

Evaluation of Stereoscopy and Lit Shading for a Counting Task in Knot Visualization

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Abstract *We present an experiment on depth cues for knot visualization in desktop virtual reality (fishtank VR). We used a within-subjects experiment to investigate stereoscopic visuals and 3D lit shading. Participants were required to count crossings in visualized knots – a fundamental task in classical knot diagram interpretation. Participants made significantly more errors with stereoscopic visuals than with monoscopic visuals, while no effect was detected for shading. Since this contrasts evaluations of fishtank VR for other applications, it provides knowledge about limitations of stereo imaging. In addition to the experiment, we outline our knot visualization approach, which uses a dynamic simulation and charged rope analogy to generate knot shape, and includes a technique for rendering gaps like those found in classical knot diagrams.*

Keywords: Knot Visualization, Stereoscopy, Fishtank Virtual Reality, Counting Tasks, Dynamic Simulation

1 Introduction

We present a study of depth cues (stereoscopy and lit shading) for knot visualization in fishtank (desktop) virtual reality. Informally, a knot can be described as a tangled piece of string, and knot visualization can help knot theorists and other users explore knot-related concepts. To illustrate, an example image from our visualization system can be seen in Figure 1.

Several knot visualization software packages, e.g., [12], [14], have been developed for interactive visualization of knots, and researchers have expressed a desire to increase the use of virtual reality techniques for knot visualization. However, we are not aware of any existing work assessing the impact of depth cues, or virtual reality in general, in knot visualization. Stereoscopy, lighting, and motion parallax have been shown to increase perfor-

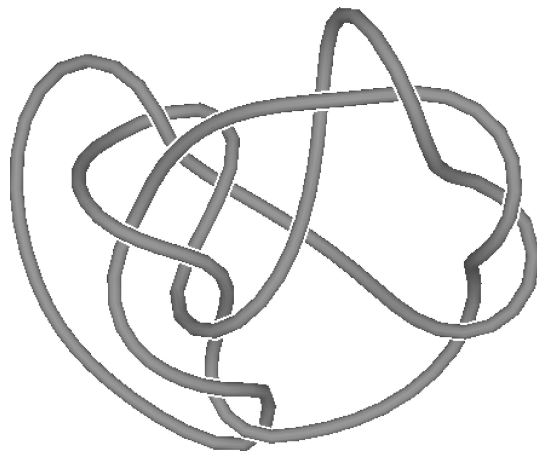


Figure 1: A knot rendered with shading and gaps.

mance and accuracy in other tasks, e.g., by [6], [7], [10], [11], [15], and [16]. In contrast to these studies, our work shows that, for a fundamental counting task, stereoscopic rendering not only fails to be beneficial, but actually reduces user performance. This result, and our later discussion, give insight into the limitations of stereoscopic viewing and the relationship to task type.

In addition to our study of depth cues, we highlight some differences between our knot visualization methods and earlier results. We include a method for rendering gaps to emphasize crossings as found in traditional knot diagrams, and we describe how a force calculation in a dynamic simulation component uses a different distance metric than previous knot visualization packages.

2 Background

The field of Knot Theory in Mathematics deals with questions such as: “Given two knots, is one deformable into the other?” Traditionally, such questions were investigated using 2D knot diagrams

with under- and over-crossings indicated by breaks (gaps) in the curve. Crossings illustrated in these diagrams are required to only involve two parts of the knot to avoid ambiguity. An extension of the classical techniques into computer-based visualization and 3D rendering may be beneficial when the complexity of the knot becomes high and the number of crossings becomes unmanageable for traditional approaches.

Existing research on knot visualization includes [2], [4], [12], and [14]. The software packages in [12] and [14] contain mechanisms for visualizing knots in three and four dimensions with differing physical and rendering attributes. In [2], a method is used to visualize the self-contact of a thickened knot. This work employs a non-linear optimization with objective function in terms of the knot’s diameter and constraints representing the knot’s geometry. As a result, several interesting metrics in knot theory can be illustrated in an interactive system. Hanson and Cross [4] have developed an illumination model for four-dimensional objects that includes shading and occlusion. These algorithms provide interactive quality rendering of tessellated 3-manifolds and can be implemented on current graphics hardware.

The importance and effectiveness of stereopsis has been evaluated extensively in the literature. Some pertinent examples are [7], [11], [15], and [16]. A “fishtank” system, defined by the use of stereo viewing and head tracking in a desktop configuration, was initially evaluated by Ware [15]. Stereo head-tracked feedback was shown to improve both subjective impressions of simple 3D environments and error data for users tracing tree paths with overlapping branches. In a later work by Ware [16], the effect of stereopsis and rotating motion on the understanding of 3D graphs was investigated. The task involved finding short paths, of length two or three, between random nodes in a fixed time period. Results indicated that the best performance occurred under stereo and motion. As our experiment deals with the user’s understanding of crossings, one may expect similar findings.

Examples of other tasks found to benefit from stereo include placing an object in a scene and estimating distance in a scene [6]; resizing objects relative to others in a scene [7]; touching the top of an object in a scene [10]; and counting shapes of objects, counting sizes of objects, determining relative densities of volumes, and determining connectivity of a volume [11]. Thus, one may expect stereo to increase user performance for a wide range of tasks. At the same time, potential limits and problems with stereoscopic visuals have been sug-

gested in [13] [17]. We discuss these further after presenting experiment results.

3 Visualization Techniques

We generated images of knots using a dynamic simulation of a charged rope, as suggested in [3]. This simulation created knots similar in appearance to output of packages such as KnotPlot [12] and Seifert View [14]. However, output differs slightly from these packages in that we add small visual gaps to emphasize crossings as in traditional knot diagrams, and our dynamic simulation uses a different distance metric for computing electrostatic forces between knot segments. Note that, although generation of knot images involves a dynamic simulation, images presented to experiment participants were static.

The charged rope model represents a knot as a set of nodes (particles) along the knot’s path. Given a set of node coordinates representing a knot, the knot image is drawn by extruding a polygonal cross section along the knot path defined by the nodes, using reference frames as in [1]. The positions of the nodes are controlled by the dynamic simulation.

3.1 Gap Rendering Technique

The classical style of a knot diagram calls for a parallel projection of a three dimensional knot with gaps in the drawing to indicate under and over crossings [8]. We render gaps in our 3D model using a two-pass method. Letting d be knot thickness and c some constant above 1, the passes are:

Pass 1:

1. Enable front side culling and disable the color buffer.
2. Draw the knot with a thickness $c \cdot d$ (drawing occurs only to the depth buffer).

Pass 2:

1. Disable front side culling and enable the color buffer.
2. Draw the knot with thickness d .

This produces an expanded hemi-cylinder (first pass) into which the usual knot is drawn (second pass), resulting in images such as Figure 1. For illustrations in this paper and for experiment images, $c = 1.33$.

3.2 Dynamic Simulation

In the dynamic simulation that generates knot shape, particles from a charged rope model [3]

are connected to their neighbors by virtual spring-dampers and are repelled by other knot segments based on simulated electrostatic forces. We use a public domain implementation of Fourth Order Runge-Kutta [9] to integrate the resultant forces. What follows are the main components of our dynamic simulation:

3.2.1 Spring-Damper Force

For each pair of adjacent particles, a length-preserving force following Hooke's Law is implemented. A force proportional to the change in distance (relative to an original, or rest, length) is applied to each pair of adjacent particles. A damping force is added proportional to relative velocity between two adjacent particles.

3.2.2 Viscous Drag

To stabilize overall knot motion, a viscous drag force is introduced to simulate a fluid of arbitrary thickness in which the simulation occurs. This is a damping force proportional to current velocity on a per-particle basis.

3.2.3 Electrostatic Repulsion

From Coulomb's Law, electrically charged particles of like polarity repel one another in proportion to the inverse square of separation distance. Accordingly, we can apply forces based on the distance between each pair of knot nodes. However, when knot segments are close to each other, we treat them as line segments for calculating the distance between them.

Point Distance The point-based distance model is sufficient for distant segments, and is similar to the model used in other knot visualization work. In this model, distance for computing electrostatic forces is computed for each pair of nonadjacent particles. In other words, the charged rope is treated as a set of charged particles rather than a charged continuous structure.

Line Distance For knot segments that are near to each other, we use a line-segment-based distance measure to reduce problems of segments passing through each other (such problems are ignored or treated kinematically in other knot visualization work).

To compute distance between line segments, we define each section of the knot model as a parametric line segment (the segment is between adjacent

particles). Distance formulae for two segments were derived by viewing the problem as constrained optimization of a distance function in terms of line equation parameters, as in [5]. For each segment, force is distributed between two particles using weights based on the parameter values corresponding to minimum distance.

Regarding the decision for use of the line distance model versus the point distance model: the decision is based on a threshold by comparing point distance to the squared sum of line segment lengths.

4 Experiment

We conducted an experiment to evaluate the effect of depth cues on user performance in a fish-tank virtual reality system for knot visualization. Specifically, we investigated stereo rendering and lit shading for a task that required users to count knot crossings from a fixed view point.

4.1 Design

A within-subjects experiment was conducted over one independent variable concerning visual feedback. The four levels of the independent variable were:

1. MF: monoscopic (zero eye separation) and flat (monochrome shading)
2. MS: monoscopic and shaded (lit shading)
3. SF: stereoscopic and flat
4. SS: stereoscopic and shaded

Examples of each case can be seen in Figure 2. The illustrated knot, as well as all others, were chosen to ensure that left, right, and center images had consistent crossing counts. (Otherwise, it is possible for two different knot projections to have two different crossing counts, and this is also a reason for the use of a fixed viewpoint in our experiment).

The dependent variable was user error rate, defined as the number of incorrect user responses about crossing count. The task required users, for each trial, to count crossings visible in a knot image displayed for a fixed period of time (8 seconds). A total of 80 trials were used per subject, divided into 4 subsets of 20 trials each (20 trials per condition). The order of the 80 images was fixed for all subjects, but the order of conditions was varied per subject, with each of the possible 24 orders being used once (24 total subjects). So, specific knots were not tied to specific conditions and order effects were addressed.

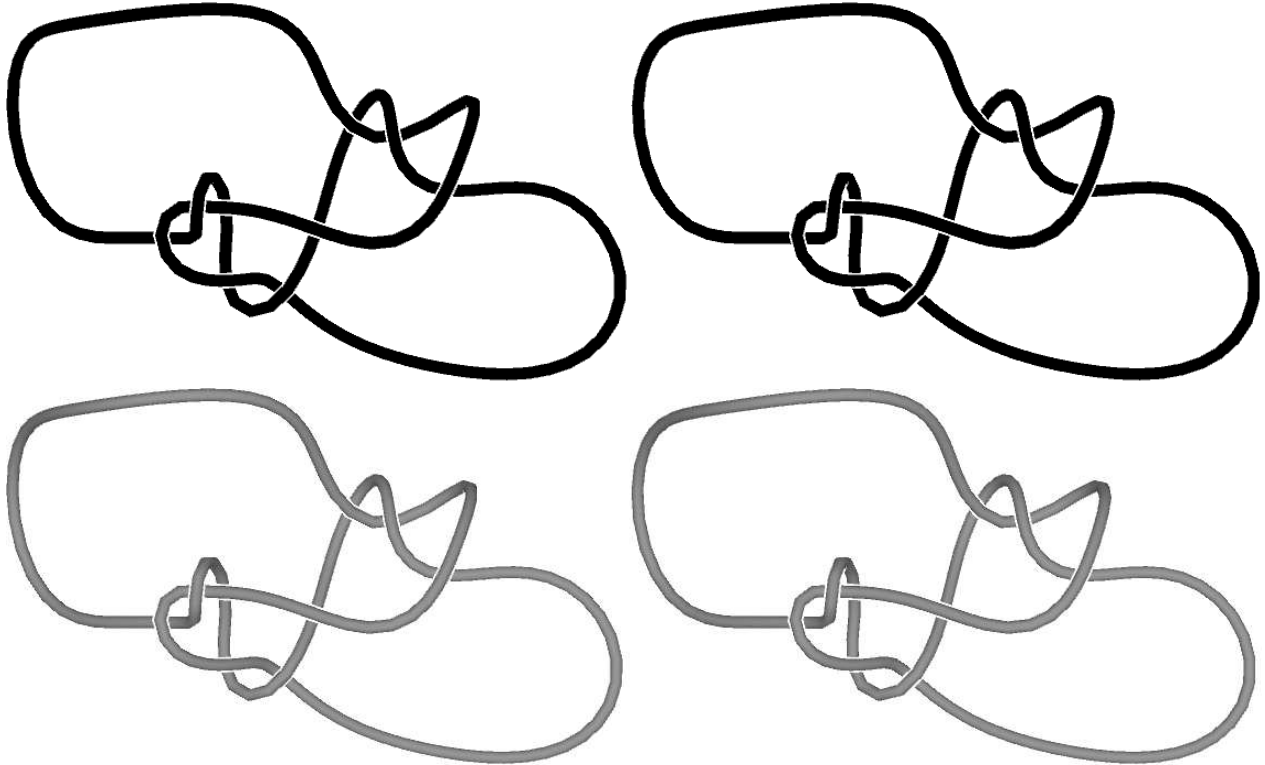


Figure 2: TOP ROW: A stereo pair of a knot representing the SF case. BOTTOM ROW: stereo pair of a knot representing the SS case. These pairs can be viewed with cross-eyed stereo free viewing (right eye image on left, left eye image on right). Viewing only one image from a pair gives a monoscopic view.

Our research hypothesis was that the addition of stereo and lit shading each impact user performance.

4.2 Materials

The glasses worn by each participant throughout the entire experiment were a pair of Crystal Eyes Workstation active stereo glasses. A standard PC was used to control and display the experiment. The monitor was a Sun Microsystems 21" CRT (model GDM-5410), set at 1024×768 resolution with a 120 Hz refresh rate to allow adequate stereo refresh rates (60 Hz per eye). To avoid effects of conditions not under investigation, and to avoid changes in crossing count that can occur with changes in projection, no head tracking was used. The user was kept at the correct eye-point, one meter from the monitor, throughout the experiment and not allowed to point at the screen during counting. Stereo images were rendered on the monitor such that the center of the 3D knot was on the mon-

itor plane, to ensure a reasonable accommodation cue (eye focus close to real device plane). We set eye separation to 85 mm, and the room in which the experiment took place was darkened. All viewing parameters were chosen for good stereo viewing based on their similarity to real viewing geometry and based on the experiences of the investigators and pilot users.

4.3 Participants

There were a total of 24 participants in the experiment, consisting entirely of university students and staff, most being students in undergraduate and graduate computer graphics courses. Three were female and the rest were male. None had used the system before.

4.4 Procedure

Before the experiment, each participant was asked to read and sign a consent form. Once completed, the stereo glasses were donned and a demonstration of the interface was given. The demonstration illustrated a knot, a graphical timer bar, and an automated question/answer text, as seen in Figure 3. To prepare each participant, they were asked if they understood the image on the screen. The investigator confirmed that each participant was able to

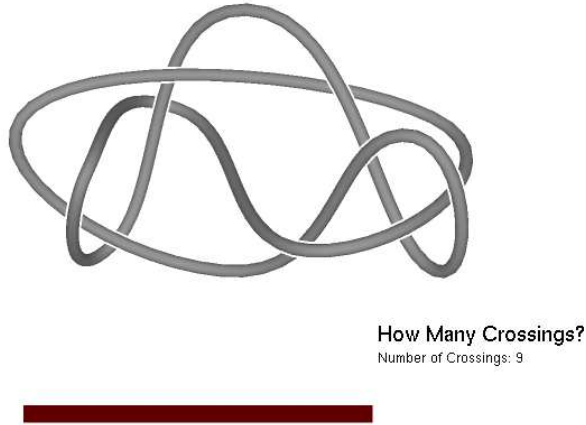


Figure 3: A demonstration of the experiment interface and protocol; timer and text placed together for brevity.

point out the fact that portions of the knot crossed over other portions. The participant was then told how many crossings were in the demonstration knot and asked to confirm it.

In the particular projection used for the demonstration, the image showed more crossings than the minimal crossing count for the knot (minimum number possible when considering all possible 3D rotations). This was intentional, to help demonstrate that crossing count was to be based only on the projected image (even without such a demonstration, it is unlikely that participants would know enough about 3D knots to consider performing the more difficult task of determining minimal crossing count).

Once familiar with the system and task, the participants proceeded with the trials. In each trial, a knot was displayed for eight seconds. The timer bar shown at the bottom-left of Figure 3 counted down the time. Once time expired, the knot was cleared from the screen and the prompt at the bottom-right of Figure 3 appeared. Users replied with a count and were not timed for the response.

At the end of each trial, after the subject indicated count, the correct count was displayed to give immediate feedback and help reinforce successful counting.

4.5 Results and Analysis

A box-and-whiskers plot summarizing error rates for all conditions is shown in Figure 4. Pairwise Wilcoxon Signed Rank Tests for Related Samples were performed, with Bonferroni adjustment of p-

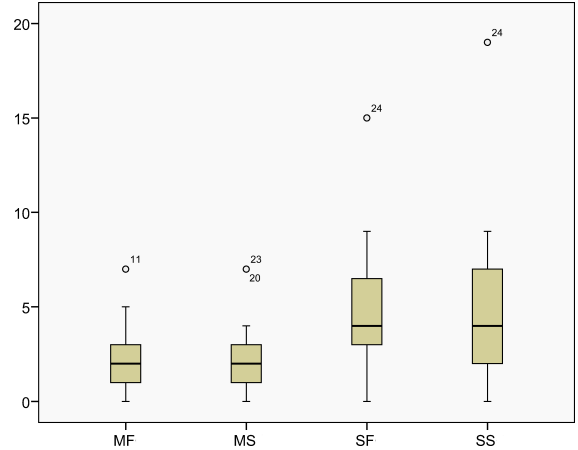


Figure 4: Box-and-whiskers plots of error rates for conditions MF, MS, SF, and SG. Boxes and whiskers show quartile boundaries, excluding outliers that are shown by points.

Table 1: Pairwise Wilcoxon p-values after Bonferroni Adjustments (after multiplication by 6, but limited to atmost 1.0).

	MF, MS	MF, SF	MF, SS	MS, SF	MS, SS	SF, SS
<i>p</i> -values	1.00	0.00	0.01	0.00	0.03	1.00

values.¹ Adjusted p-values are shown in Table 1.

As seen in the plot and table, we observed an adverse effect of stereo on user performance. For all cases involving stereo, a significant decrease in performance was detected ($p < 0.05$) when compared to any monoscopic case. The adjusted p-values of MF versus SF, MF versus SS, MS versus SF, and MS versus SS were 0.00 ($Z = -3.684$), 0.01 ($Z = -3.142$), 0.00 ($Z = -3.684$), and 0.03 ($Z = -2.804$), respectively.

No effects were detected by comparisons between lit and monochrome (flat) shading (MF versus MS, SF versus SS).

4.6 Discussion

The experiment demonstrated not only a failing of stereoscopic visual cues to aid in the task of counting crossings, but a harmful effect of stereo. This may surprise readers familiar with other evalua-

¹Some readers may prefer to see the four conditions treated as a combination of two bilevel factors. A Two-Way ANOVA for related samples was also performed. It reported equivalent findings of significance and no detected interaction effects. ANOVA results are not detailed here because its assumptions are not met.

tions of stereo, including those cited earlier in this paper. The task of counting crossings may not benefit from stereo if crossings are easily identified in a 2D projection with rendered gaps. However, this does not explain the reduction in performance with stereo. There are several possible aspects of stereo viewing that may negatively impact performance, but further work would be needed to evaluate them.

To speculate briefly, one possible reason is that stereo viewing requires additional control over eye convergence (rotation) for viewing object regions at different virtual depths. At the same time, there is a conflicting accommodation cue (eye focus) because eye focus remains on a fixed plane unlike it does in a real 3D environment (a similar conflict occurs in HMD and projection devices for VR). As discussed in [13], this disparity can result in discomforting visual stress for the user and can increase with enhancements to image resolution. Additionally, a ghosting effect encountered with stereo glasses has been shown to exacerbate the stress and fusion efforts when combined with convergence activity over prolonged periods [17]. Finally, the added depth cue may increase cognitive load on users if it causes them to form a more elaborate mental model than the monoscopic view.

It remains possible that stereo would be beneficial for other tasks in knot visualization, such as 3D interaction with knots, or even variations of the counting task that rely increasingly on 3D, such as distinguishing under- and over-crossings or performing mental rotations to determine minimal crossing count under all orientations. However, in light of our study, the use of stereo for knot visualization would require justification beyond that of existing studies from other applications.

4.6.1 Participant Feedback

During the experiment, the investigator noted any comments made freely by subjects. Subjects also freely made comments after the experiment, during debriefing. In a post-experiment, informal question, participants were encouraged to describe any trouble they had with specific conditions. Some subjects indicated that stereo seemed distracting for the counting task. The gaps (rendered for each case) were noted by several participants as effective landmarks.

A non-trivial torus knot presented to participants is shown in Figure 5. 43% of participants counted this knot incorrectly and, of these, some reported losing track of their starting position. Here, the common problem may be its cyclical nature.

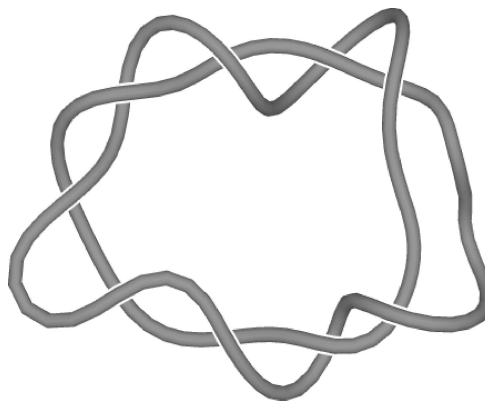


Figure 5: This torus knot was miscounted by 43% of participants.

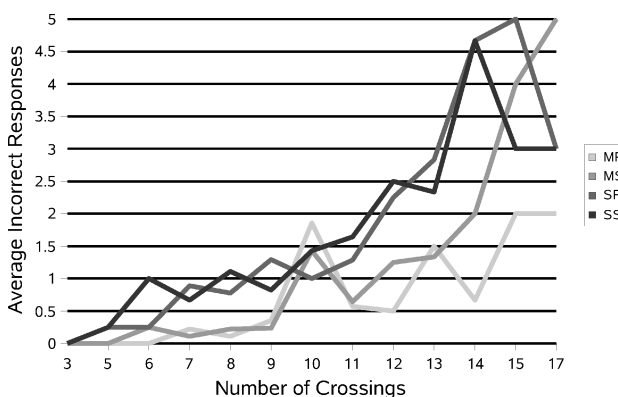


Figure 6: This chart shows a plot of error rates against number of crossings.

4.6.2 Noteworthy Trials

We inspected error rates for specific trials. Only four produced incorrect answers for at least half of all responses. Of these, only one was missed by more than 60% of all participants. A plot of average error rate against number of crossings is given for each condition in Figure 6. A trend of lower performance with stereo is seen across most knot complexities.

In Figure 7, the knot with the highest number of crossings, seventeen, is shown. Several participants reacted audibly at the appearance of this knot, expressing concern. The error rate for this knot was 54%, the third highest in this study.

5 Conclusion

We presented a study of stereoscopy and lit shading for knot visualization. We showed that stereo reduces user performance for the elementary task



Figure 7: Several users reacted audibly to this knot. It was missed by 54% of participants. It has been rotated 90° clockwise to conserve space.

of counting crossings. No effect was detected for lit shading compared to monochrome shading. Stereopsis, while an important depth cue in many virtual environments, does not always yield optimal results. Our study helps give insight into its limitations.

For future work, we have already suggested experiments based on related tasks that require increasing use of 3D information (distinguishing under- and over- crossings, determining minimal crossing count under all rotations, or distinguishing between multiple knots). Such experiments may give further insight into the limitations of stereo illustrated by our present work. Other virtual reality technologies, such as head tracking and 3D interactions tools, should also be considered for full evaluation of 3D interfaces for knot visualization.

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